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A Review on Affine Transformation

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Abstract: The growth of multimedia applications, Fractal image compression techniques is very important for the efficient transmission and storage of digital data. The purpose of this paper, we described fractal image compression using affine transformation correspond to translations, scaling, rotations and reflections in metric space.

Keywords: Fractal image compression, affine transformation, image coding and decoding.

1. INTRODUCTION

In geometry, an affine transformation in latin "affinis" means "related" is a transformation introduced by Euler in 1748 in his book "Introduction in analysis in infinitorum then 1827 August Mobius wrote on affine geometry.
In geometry, an affine transformation is a function that maps an object from an affine space to another and which preserve structures. Indeed, an affine transformation preserves lines or distance ratios but change the orientation, size or position of the object. The set of affine transformation is composed of various operations. Translations which modify object position in the image. Homothetic transformations composed of the contraction and dilatation of an object, both scaling operations. The transvection (shear mapping) which shifts every point of an object. Rotation which to allows to rotate an object according to its axis[3]. Affine transformation improved of compression ratio of fractal image compression. The improvement of compression rates is done by applying the lossy compression techniques on the parameters of the affine transformation of the fractal compressed image.

2. AFFINE TRANSFORMATION

The use of homogeneous coordinates is the central point of affine transformation which allow us to use the mathematical properties of matrices to perform transformations. So to transform an image, we use a matrix $T \in M_3(R)$ providing the changes to apply

$$T = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ p_x & p_y & 1 \end{bmatrix}$$

The vector $[T_x, T_y]$ represent the translation vector according the canonical vectors. the vector $[P_x, P_y]$ represents the projection vector on the basis. The square matrix composed by the a_{ij} elements is the affine transformation matrix [3], [5]. An affine transformation $T : R^2 \rightarrow R^2$ is a transformation of the form $T : Ax + B$ defined by

$$T\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ 1 \end{pmatrix}$$

Where the parameter $a_{11}, a_{12}, a_{21}, a_{22}$ form the linear part which determines the rotation, skew and scaling and the parameters t_x, t_y are the translation distances in x and y directions, respectively[1], [2].

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = T\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x\,a_{11} + y\,a_{12} + t_x$$
$$y' = x\,a_{21} + y\,a_{22} + t_y$$

The general affine transformation can be defined with only six parameter:

θ : the rotation angle.
t_x : the x component of the translation vector.
t_y : the y component of the translation vector.
S_x : the x component of the scaling vector.
S_y : the y component of the scaling vector.
Sh_x : the x component of the shearing vector.
Sh_y : the y component of the shearing vector[3].

In the other words, The Fractal is made up of the union of several copies of itself, where each copy is transformed by a function T_i, such a function is 2D affine transformation, so the IFS is defined by a finite number of affine transformation which characterized by Translation, scaling, shearing and rotation.

2.1 Translation

A translation is a function that moves every point with constant distance in a specified direction by the vector $T = [T_x, T_y]$ which provide the orientation and the distance according to axis x and y respectively[3].

The mathematical transformation is the following[4], [6]:

$$T' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x + t_x$$
$$y' = y + t_y$$

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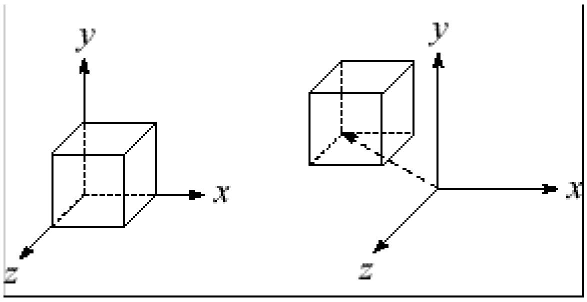


2.2 Transvection

A transvection is a function that shifts evry point with constant distance in a basis direction x or y[3].

The mathematical vertical and horizontal transvection respectively is the following[4], [6]:

$$T' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

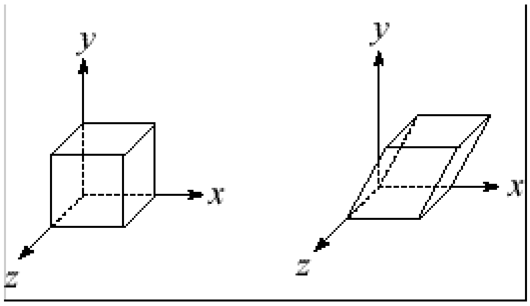
$$x' = x + y\, s_v$$

$$y' = y$$

$$T' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x$$

$$y' = x\, s_h + y$$



2.3 Rotation

Rotationis a circular trnsformation around a point or an axis. A rotation around an axis defined by the normal vector to the image plane located at the center of the image[3].

The mathematical rotation about clockwise and counter clockwise is the following[4], [6]:

$$T' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

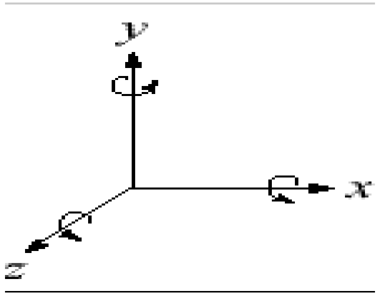
$$x' = x\, cos\theta + y\, sin\theta$$

$$y' = -xsin\theta + y\, cos\theta$$

$$T' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x\, cos\theta - y\, sin\theta$$

$$y' = x\, sin\theta + y\, cos\theta$$



2.4 Scaling

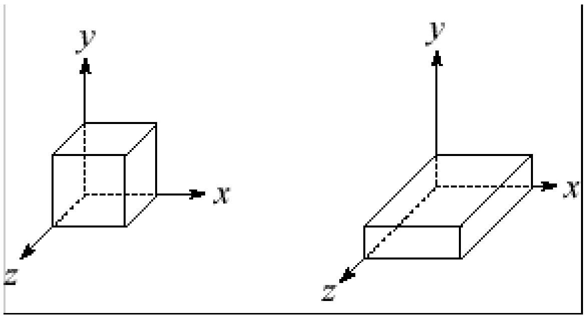
Scaling is a linear transformation that enlarge or shrinks objects by a scale factor that is the same in the direction[3].

The mathematical transformation is the following[4], [6]:

$$T' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x\, s_x$$

$$y' = y\, s_y$$



REFERENCES

[1] Barnsely M. F., " Fractal Everywhere", Acadmic Press, San Diego, 1993.
[2] Barnsely M. F. and Hurd L.P., " Fractal Image Compression", AK Peters Ltd, Wellesely, Massachuretts, 1992.
[3] Coste, "Affine Transformation, Landmarks registration", Nonlinear warping, October, 2012.
[4] David F. Rogers and J. Alan Admas, "Mathematical elements for computer graphics", 2nd Ed, NewYork, 1990.
[5] Yan Bin Jia, "Transformaions in Homogeneous coordinates", Notes 2014.
[6] University of Texas at Austin, coputer graphics, 2010.